The assessment of viscoelastic models for nonlinear soft materials

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Abstract— **The increasing use of soft materials in robotics applications requires the development of mathematical models to describe their viscoelastic and nonlinear properties. The traditional linear viscoelastic models are unable to describe nonlinear strain-dependent behaviors. This limitation has been addressed by implementing a piecewise linearization (PL) in the simplest viscoelastic model, the Standard Linear Solid (SLS). In this work, we aim to implement the PL in a more complex model, the Wiechert model and compare the stress response of both linearized models. Therefore, the experimental data from the stress relaxation and tensile strength tests of six rubber-based materials is used to approximate the spring and dashpot constants of the SLS and the Wiechert model. Prior to implement the PL into the stress-strain curve of each material, the stress response from the Maxwell branches must be subtracted from this curve. By using the parameters obtained from fitting the Wiechert model into the stress relaxation curve, the response of both linearized models was improved. Due to the selection of constitutive equations evaluated, the linearized SLS model described the stress-strain curve more accurately. Finally, this work describes in details every step of the fitting process and highlights the benefits of using linearization methods to improve known models as an alternative of using highly complex models to describe the mechanical properties of soft materials.**

I. INTRODUCTION

The implementation of soft materials in robotics applications has always been challenging [1]. This is the case of elastomers that, commonly used in soft robotics, exhibit nonlinear, time-dependent and history-dependent properties which cannot be easily described by mathematical models. However, the benefits provided by soft materials such as energy storing, passive compliance and safe human-robot interaction, have motivated their implementation in robotic applications and the development of mathematical models able to describe them [2].

In robotics applications for human assistance, mimicking the human musculoskeletal system and its natural properties of storing and releasing energy have motivated the inclusion of elasticity into these applications. Series-elastic actuators (SEAs) are commonly implemented to achieve the latter. The addition of an elastic element between the actuator and the load greatly simplifies the controller design. The deformation of the elastic element can provide an indirect measurement of the applied force to the load, essentially transforming a force-control problem into a displacement-control problem [3]. Traditional SEAs use metallic springs, considered as purely elastic. However, the human musculoskeletal system exhibit viscoelastic behaviors. Viscoelasticity have proven beneficial in these particular applications, in terms of energy consumption and compliance, motivating the implementation of viscoelastic (soft) materials in SEAs [4], [5], [6]. The mechanical behavior of a rigid element as a spring can be accurately described by known mathematical models. This is not the case for soft materials which have nonlinear and viscoelastic properties. The mentioned benefits can only be fully exploited by developing a mathematical model able to accurately describe the mechanical behavior of soft materials.

Substantial research is focused on developing new models suitable for the latter task. The most accurate models are mathematically complex and computationally expensive [7], [8], [9]. Nonetheless, even these complex models cannot account for all the different factors which modify the materials properties, such as the manufacturing process and internal weakening of the material after being loaded for the first time [10]. The latter highlights the difficulty of developing mathematical models which account for both microscopic and macroscopic aspects of the materials. This is why some authors have opted for alternative methods for characterizing a material, such as Finite Element Analysis.

In robotics applications, where the controller can compensate the lack of accuracy in describing the controlled plant, a simple and fairly accurate model is preferred over very accurate and highly complex one. In line with this, the work performed by Austin et al. modifies the viscoelastic Standard Linear Solid (SLS) model by implementing a piecewise linearization (PL) which allows the model to account for nonlinearities in the material and improve its accuracy without highly increasing the model complexity [6], [11]. Whether the aim is to develop a highly accurate model or a fairly accurate one, both approaches make use of the basic linear viscoelastic models, illustrated in Fig. 1.

In this work, the analysis and comparison of two viscoelastic models when describing the properties of soft materials is presented. The PL described in the literature is applied to the Standard Linear Solid (SLS) model and to the Wiechert

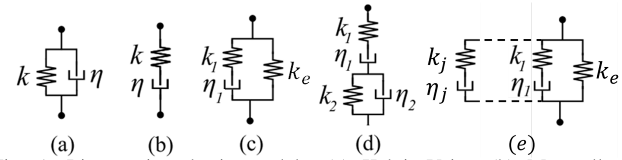

Fig. 1: Linear viscoelastic models. (a) Kelvin-Voigt. (b) Maxwell. (c) Standard Linear Solid. (d) Burger. (e) Wiechert. k and η represent the spring's stiffness and the dashpot's viscous constant, respectively [11].

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model, a more complex version of the SLS model. The potential benefits of implemented the PL in a more complex model are of interest. In contrast with the available literature, we use the stress relaxation test and the respective Wichert model's equation to characterize the spring stiffness k and dashport viscous constant η of all the soft materials. Following the premise that any material can be fully described with the Wiechert model, given enough Maxwell branches, we find the optimal number of branches between the range of 1 to 10 which better approximate each material. Prior of implementing the PL on the models, we account for the stress contribution of the Maxwell branches and remove it from the tensile strength test data. This additional step greatly improves the stress response of the models. Subsequently, the strain-dependent stiffness is extracted from the stress-strain curve by implementing the PL. Lastly, the constant spring stiffness found in the traditional model is substituted by the obtained strain-dependent stiffness and the models stress response is evaluated.

II. METHODOLOGY

A. Viscoelastic Models

Rubber-based materials are known to have a nonlinear relationship between the applied strain and the resulting stress, as well as time-dependent and history-dependent properties. The linear viscoelastic models, which are based on different configurations of mechanical elements such as springs and dashpots, (Fig. 1) can describe the latter properties. In these models, the parameters of interest are the springs stiffness k and the dashpots viscous constant η.

In particular, the SLS model is frequently used when modeling viscoelastic materials, mainly due to its fairly simple mathematical model and its ability to account for creep and stress relaxation of the material (time-dependent properties). Creep in a material refers to the change over time of the strain under a constant stress. Similarly, the stress relaxation of a material refers to the change over time of the stress under a constant strain. The SLS model can be viewed as a Maxwell model (also known as Maxwell branch) with an extra spring connected in parallel.

The simplicity of the SLS model is also its main limitation. Viscoelastic materials are known to have more than one relaxation time, i.e. more than one Maxwell branch. In the linear viscoelastic models, the relaxation time depends on the viscous elements, i.e. dashpots. The Wiechert model, which is essentially a SLS model with j Maxwell branches is able to account for j relaxation times. The time-dependent behavior of any viscoelastic material can be fully described by this model, given enough numbers of elements. However, the complexity of the model increases in proportion to the number of extra branches. Mathematically, each extra branch increases the derivative order of the model since more equations are required to account for the extra variables [12], [13]. Viscoelastic materials also have history-dependent properties. The cross-linked molecular chains found in viscoelastic materials relocate when the material is deformed for the first time. This process is known as the Mullins effect which causes the material to permanently lose strength [10].

In addition to time-dependent and history-dependent properties, rubber-based materials also have a nonlinear stress-strain relationship which is partially described by the linear viscoelastic models. The relaxation time of the dashpots in these models allows nonlinear stress responses to be described. This is a time-dependent nonlinearity. Nonetheless, viscoelastic materials also have a nonlinear strain-dependent response which can be described by implementing the PL proposed in [6], [11]. The spring in parallel with the other elements, in both the SLS model and the Wiechert model, is known as the equilibrium spring and its stiffness k_e is assumed constant. However, the stiffness k_e of a material is not constant and also depends on the material's strain.

Early attempts of modeling a strain-dependent stress response in viscoelastic materials are described by Schepelmann et al. in [6], where the stress-strain curve of a nonlinear rubber spring is approximated with an exponential model. In subsequent works, Austin et al. describes a piecewise linear regression fitted to the stress-strain curve of a material. The work is based on the SLS model. The slope of the stress-strain curve represents the material's Young Modulus which is proportional to the materials' stiffness. During a tensile strength test the material is deformed at a constant rate, i.e. the stress response of the viscous element is also constant. Therefore, the observed nonlinearities in the stress-strain curve are caused by the equilibrium spring. The PL approximate the nonlinear behavior of the equilibrium spring by considering it as several springs in parallel which "engages" in sequence as the materials strain increases. This is modeled by a summation of Heaviside functions centered in the desired strain in which each of the mentioned springs "engages" and contributes with the total stress response of the material. In other words, the stress-strain curve of the material is segmented in several sections which relates a single stiffness to a strain range (Fig. 2).

B. Experimental work

In order to describe a viscoelastic material using the above models, it is required to obtain the materials parameters from experimental data. The mechanical tests of tensile strength (ASTM D412 [14]) and stress relaxation (ASTM D6147 [15]) were performed for these soft materials: ethylene polypropylene rubber (EPR), fluorocarbon rubber (FR), nitrile rubber (NR), natural rubber (NatR), polyethylene rubber (PR) and silicone rubber (SR). The samples were obtained from a material sheet using laser cutting, a

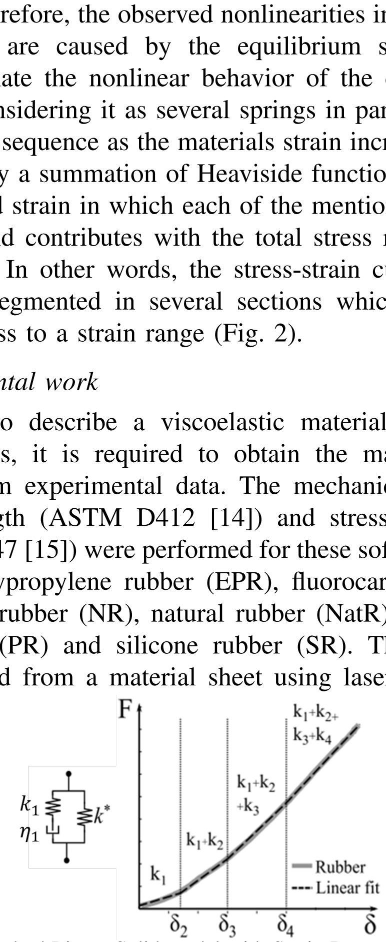

Fig. 2: Left: Standard Linear Solid model with Strain-Dependent Stiffness. Right: Piecewise linearization method [11]. This load-deformation (δ) curve is proportional to a stress-strain curve.

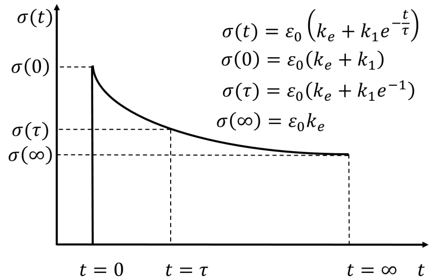

Fig. 3: Typical stress relaxation curve. The parameters k_e, k_1 and η can be obtained by analyzing three points in the curve: $t = 0$, $t = \tau$, and $t = \infty$.

technology commonly used with rubber-based materials. All the materials have a thickness of 1.5 mm, except for the polyethylene rubber in which the thickness is 6 mm. All tests were carried out using an Instron 3369 Dual Column Testing System equipped with a 50 kN load cell.

In the tensile strength test, all the specimens were elongated to failure at a deformation rate of 500 mm/min except for silicon rubber and natural rubber which were elongated at 50 mm/min. The machines gripper was unable to hold the latter materials using the highest deformation rate.

During the stress relaxation test the materials were elongated at a deformation rate of 500 mm/min until reaching a predefined initial strain ϵ_0. The initial strain ϵ_0 for each material was selected using its stress-strain curve, ensuring that the strain value was within the elastic region of each material (on average, less than 0.8 strain). The latter condition is to avoid incorrect measurements caused by plastic deformations in the material. The materials were held at ϵ_0 for a period of three hours. The duration of the experiment was taken from similar experimental works on soft materials [10], [16]. Prior to the actual model fitting, the obtained experimental data from the five specimens per material was compiled into a single dataset and subsequently smoothed using a weighted second degree local regression in MATLAB.

C. Model fitting

The mathematical expression for the SLS model and the Wiechert model can be simplified when considering a constant strain input (stress relaxation test). This simplification allows these models to be fitted into the stress relaxation curve and to approximate the parameter of interest, k and η [13]. The mathematical expression for the Wiechert model under a constant strain input is given by:

$$\sigma(t) = \left\{ k_e + \sum_j k_j e^{-t/\tau_j} \right\} \epsilon_0 \qquad (1)$$

where σ is the stress at a given time, k_e is the equilibrium spring stiffness and ϵ_0 is the initial strain. For the summation, $\tau_j = \eta_j / k_j$ is the relaxation time constant, k_j and η_j are the spring stiffness and viscous constant, respectively, of the elements in the j^{th} Maxwell branch. For the specific case when $j = 1$, the resulting equation describes the SLS model under a constant strain input. In this case, the three parameters described in the SLS model: equilibrium spring stiffness k_e, dashpot viscous constant η and the spring stiffness in the Maxwell branch k_1, can be obtained from the

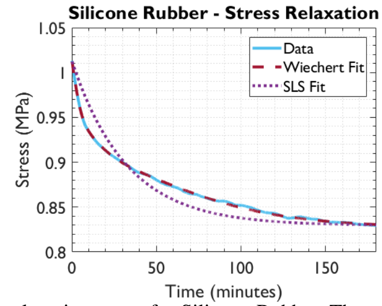

Fig. 4: Stress relaxation curve for Silicone Rubber. The number of branches used in the Wiechert model fit is $j = 10$.

stress relaxation curve by analyzing three significant points: $t = 0$, $t = \tau$, and $t = \infty$ (Fig. 3). The longer the duration of the test, the better the approximation of k_e.

The process to extract the parameters of the Wiechert model is more complicated due to its extra Maxwell branches, i.e. there are more than three points in time to be analyzed. These points can be selected using a collocation technique [13], [17]. In the reviewed literature, the points of interest are linearly scattered throughout the whole duration of the stress relaxation curve. Nevertheless, the decaying exponential term in Eq. 1 is better approximated by selecting the points of interest using a logarithmic scale. This is possible with the MATLAB function `logspace` which spreads evenly the desired number of points between the allowable decades. Supposing that a Wiechert model with six branches, $j = 6$, wants to be fitted into a stress relaxation curve with four decades of duration ($t = 10^4$ seconds). In total it would be required seven points in time, one for each branch and one for $t = 0$ (curve first point) which the function texttlogspace spreads as evenly as possible. The point $t = 0$ is required for a correction described in the following paragraph. Let us focus in the branches. Each point in time represents a time constant τ_j for which there is a known stress σ_j from the experimental data. This can be rearranged into an equation system of j equations with k_j as the unknown variable as described in [17].

Prior to this step, k_e can be obtained using the equation for $\sigma(\infty)$, defined for the SLS model (Fig. 3). Subsequently, The Wiechert model in Eq. 1 can be completely described by solving the mentioned system of equations. Finally, after obtaining all the k_j, the value of k_1 is corrected, as described in [13], by analyzing the point in time $t = 0$ (Fig. 3).

The previous process allows the wiechert model equation to be fitted into the stress relaxation curve for a defined number of branches j. However, to obtain the optimal number of branches for each material, an iterative algorithm to find the smallest root mean square error (RMSE) between the Wiechert model response and the experimental data after testing different number of branches in the range of $j = [1, 10]$ is implemented. The obtained optimal number of branches for each material varied between the range $j = [8, 10]$. A higher number of branches has a meaningless improvement on the RMSE. Furthermore, beyond the number of branches $j = 20$ the Wiechert model response shows an oscillatory behavior, hence a higher RMSE. Having obtained the parameters of interest for the SLS and the Wiechert model, their stress response under a constant strain is compared against the experimental data in Fig. 4. This figure highlights the better accuracy delivered by the extra Maxwell branches in the Wiechert model in comparison to the simpler SLS model. As previously mentioned, Eq. 1 is a simplification helpful to approximate the parameters of both models but it is only applicable when the strain input is constant. The mathematical expression for the Wiechert model which describes the stress response under an unknown strain input, also called the constitutive equation, is found in [13]. Let us again consider $j = 1$ to obtain the constitutive equation of the SLS model and transform it back to the time domain as follows (for the detailed procedure refer to [13]):

$$\dot{\sigma} + \frac{\sigma}{\tau_1} = (k_e + k_1)\dot{\epsilon} + \frac{k_e \epsilon}{\tau_1} \tag{2}$$

where ϵ, $\dot{\epsilon}$, and $\dot{\sigma}$ are the strain, the strain rate and the stress rate, respectively. Notice that the previous procedure will yield into a higher derivative order equation when applied to the Wiechert model due to its extra branches. A higher number of branches will increase the model accuracy at the cost of increasing its mathematical complexity. The constitutive equation of a Wiechert model with j branches would result in a j order differential equation similar to Eq. 2. The aim of this work is to evaluate the performance of the Wiechert model when the PL is implemented to it. Therefore, solving a complex differential equation is out of the scope. Nonetheless, the Wiechert model can be evaluated by transforming it into a finite differences equation, as explained in [13], yielding the following equation:

$$\sigma^t = k_e \epsilon^t + \sum_j \frac{k_j(\epsilon^t - \epsilon^{t-1}) + \sigma_j^{t-1}}{\left(1 + \dfrac{\Delta t}{\tau_j}\right)} \tag{3}$$

where the superscript $t-1$ and t refers to values before and after a small time step Δt have passed. The response of the two viscoelastic models of interest will be compared against the experimental data from the tensile strength test.

The next step in the fitting process focuses on the tensile strength test. It is worth noticing this test the strain rate is constant, hence the resulting stress for both models (Fig. 1) is dependent on both the equilibrium spring and the Maxwell branches. At this stage of the model fitting, the parameters of the Maxwell branches in both models are known and their stress response can be calculated. The stress response of the equilibrium spring (k_e) can be isolated by subtracting the stress response of the Maxwell branches to the stress measured in the tensile strength test.

After isolating the stress response of k_e, the final step in the fitting process is to implement the PL to both models and compare their response against the experimental data. Firstly, the stress-strain curve from tensile strength test is divided into n segments. As previously explained, k_e is considered as a group of parallel springs which "engage" as the strain increases. This means, each subsequent stiffness is a combination of the ones found in previous segments of the stress-strain curve (Fig. 2). Lastly, a linear regression is applied on the stress-strain curve for the desired n strain segments to find the slope of the curve. This slope represents the stiffness of the equilibrium spring in each segment. By combining the n obtained stiffness, the stress response of the strain-dependent stiffness k_i^* is defined as follows:

$$\sigma^* = \sum_i^n k_i^* H_{\epsilon-\epsilon_i}(\epsilon - \epsilon_i) \tag{4}$$

where n is the desired number of strain intervals to fit, ϵ_i represents the strain value at which the i^{th} spring starts contributing to the stress response, the $H_{\epsilon-\epsilon_i}$ is the Heaviside or unitary step function centered at ϵ_i, i.e. the function output goes from 0 to 1 when $\epsilon - \epsilon_i = 0$. By substituting Eq. 4 into Eq. 2, the Standard Linear Solid model with Strain-Dependent Stiffness (SLSDS) is obtained [11], [6].

Linear viscoelastic models describe a nonlinear relationship (decaying exponential time relaxation) between the applied strain and the resulting stress in a material. However, they only account for a linear stress response of the equilibrium spring. In reality, the relocation of internal molecular chains causes viscoelastic materials to exhibit a nonlinear and strain-dependent stress response. This can be solved by implementing the PL into Eq. 3. The equilibrium spring stiffness k_e is replaced by the strain-dependent stiffness k_i^*, yielding the linearized Wiechert model (PL-Wiechert) in Eq. 5. Subsequently, the SLSDS model, found in [11] is transformed into a finite difference equation, yielding Eq. 6.

$$\sigma^t = \sigma^* + \sum_j \frac{k_j(\epsilon^t - \epsilon^{t-1}) + \sigma_j^{t-1}}{\left(1 + \dfrac{\Delta t}{\tau_j}\right)} \tag{5}$$

$$\sigma^t = \frac{1}{\left(1 + \dfrac{\Delta t}{\tau_1}\right)} \left[\frac{\Delta t}{\tau_1}\sigma^* + (\sigma^* + k_1)(\epsilon^t - \epsilon^{t-1}) + \sigma^{t-1} \right] \tag{6}$$

In this work, we call Eq. 6 the piecewise linearized SLS (PL-SLS) model, differentiating it from the SLSDS model found in the literature since we implemented the optimization explained previously (stress response of Maxwell branches). In summary, the experimental data from the stress relaxation test was used to obtain the parameters in the Maxwell branches of both models. The required number of branches was different per material, ranging from $j = 8$ to $j = 10$. The constitutive equation of the Wiechert model was expressed as an equation of finite differences and subsequently linearized to obtain Eq. 5 (PL-Wiechert). Similarly, the constitutive equation of the SLS model (Eq. 2) was modified in the same way, yielding Eq. 6 (PL-SLS).

III. Results and Discussion

A. Analysis of the optimal number of strain segments

The amount of strain segments and their proper collocation impacts on the PL method accuracy. In the work presented by Austin et al. there is no explanation of the criteria used to select the strain segments, only an illustration is provided [11]. In this work, we use the variation of the slope in the stress-strain curve as the selection criteria. We developed an optimization algorithm to collocate a new strain segment when the slope has varied outside a defined tolerance boundary.

Having defined the selection criteria as the variation of the slope, we tested different tolerance boundaries and observed the relationship between the number of strain segments and: 1) the desired tolerance and 2) the achievable relative RMSE, which is the absolute error divided by the mean value of the stress-strain curve (Fig. 5). On one hand, the relationship between the number of strain segments and the desired tolerance is found to be exponential. On the other hand, the achievable RMSE for both models, in general for all materials, have minimum changes above a certain number of strain segments.This highlights a design trade-off between good accuracy and high computational load of the controller due to the large number of strain segments.

The PL-SLS model benefits the most from the PL method. It delivers a good accuracy even for small number of strain segments (Fig. 5). In contrast, the accuracy of the PL-Wiechert does not improve when using higher numbers of strain segments. Moreover, in the cases for the SR and EPR materials, the accuracy gets worse as the number of strain segments increases (Fig. 5a and Fig. 5b).

The charts in Fig. 5 are useful to select a proper value for the slope variation tolerance, taking into account the previously mentioned trade-off. It can be appreciated that the optimal tolerance is different for each material and dependent on the application. For the sake of analyzing the effect of the number of strain segments in the stress response of the PL-SLS and the PL-Wiechert models we chose a tolerance value of 20% and presented the models fit in Fig. 6.

B. Analysis of model fit accuracy

In general, the stress response from the PL-SLS model outperforms the response from the PL-Wiechert model (Fig. 6). The PL-SLS model is able to accurately describe the stress-response of all the soft materials. Furthermore, it is able to achieve values of relative RMSE close to zero in four of the six rubber-based materials tested (Fig. 5b, 5c, 5d, 5e). The slightly higher relative RMSE for the SR (Fig. 5a) and PR (Fig. 5f) materials might be caused by different factors, such as the incorrect selection of the stress relaxation test parameters, i.e. the initial strain ϵ_0 and the test duration. The PR material is unable to sustain high strains without suffering plastic deformation, even with this taken into account, an even smaller ϵ_0 is recommended. Another factor could be the time collocation method. The poor selection of the points in time to analyze can affect the accuracy of the parameters extracted. The logarithmic collocation approach used here yielded a large number of branches required to describe the material, which can cause the model response to oscillate.

In the case of the PL-Wiechert model, the very low obtained accuracy might be caused by an essential difference between Eq. 5 and Eq. 6. In the latter equation, the strain-dependent stiffness k_i^* interacts with the strain and the strain

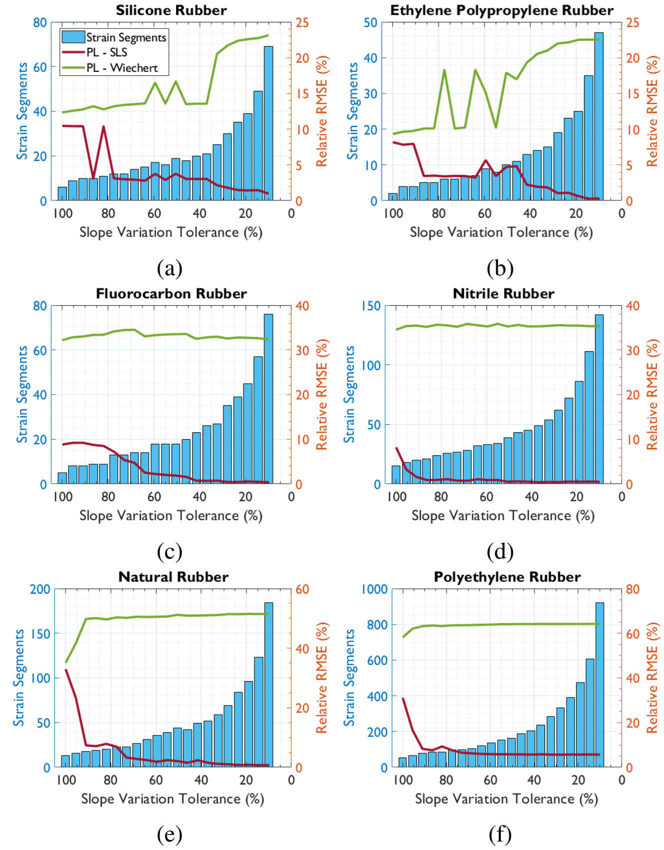

Fig. 5: Relationship of the desired tolerance between the number of strain segments (blue bars), and the achievable rRMSE of the PL-SLS (solid red) and the PL- Wiechert (solid green) models, for all the soft materials (a-f).

rate whereas in the former, k_i^* only interacts with the strain. This lack of interaction of k_i^* in the PL-Wiechert allow the abrupt step changes caused by the Heaviside function to disrupt the model stress response (Fig. 6a, 6b, 6f).

Nonetheless, the main limitation of the PL-SLS model is the inability to account the stress offset from the Maxwell branches. This was solved using the parameters obtained from the Wiechert model with j branches, ultimately improving the stress response of the PL-SLS model. The PL-Wiechert model response can be improved by using its constitutive differential equation, similar to Eq. 2. Even in its simplest form, i.e. $j = 2$ the resulting second order differential equation might outperform the PL-SLS model due to the fact that the strain dependent stiffness k_i^* will interact with different terms of the equation, providing that transforming it into a finite differences equation does not add extra complications.

IV. Conclusion

The experimental data obtained from the stress relaxation tests of six soft materials was used to describe the parameters of two mathematical models, the SLS and the Wiechert model. Both models were fit into the stress relaxation curve to extract their parameters. The fitting process for the SLS model is straight forward and the simplicity of the model yielded in a large RMSE. In contrast, the Wiechert model was fitted using a time collocation technique which yielded in a system of equations required to be solved to obtain all the

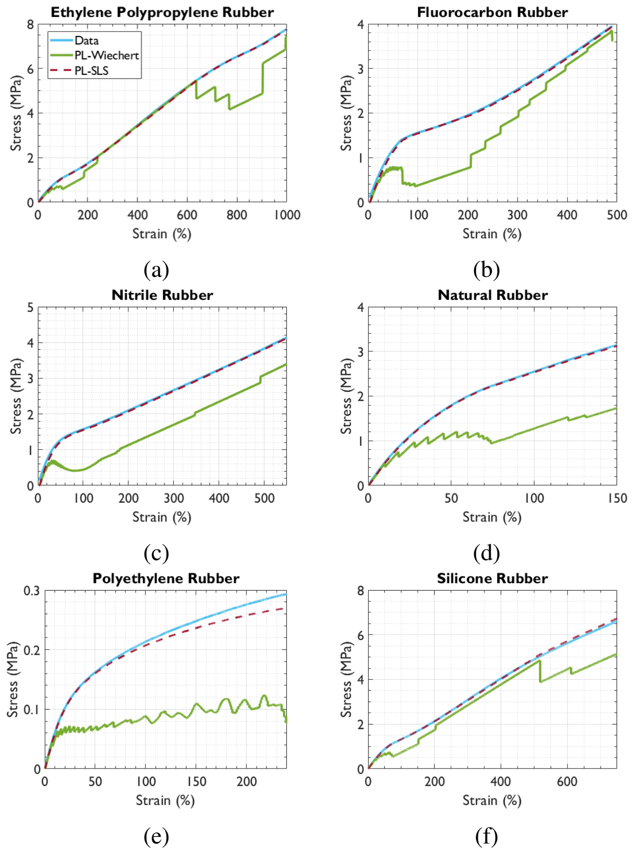

Fig. 6: Comparison between the experimental data from the Tensile Strength test and the Stress response of the PL-SLS (dashed red) and the PL-Wiechert (solid green) models for all the soft materials (a-f). The number of strain segments required to meet the slope variation tolerance of 20% for each material are EPR=20, FR=37, NatR=88, NR=73, PR=455 and SR=33.

model parameters. In addition, an optimization algorithm was implemented to obtain the right amount of Maxwell branches required to minimize the RMSE. The optimal number of branches varies from one material to the other in the range of $j = [8, 10]$. The stress response of the optimal number of Maxwell branches was subtracted from the tensile strength test data. This step allowed the piecewise linearization to better approximate the strain-dependent stiffness of the equilibrium spring. Due to the lack of explicit information regarding the PL method implementation [11], an algorithm to locate the strain segments for the PL method using the variation of the slope of the stress-strain curve as the selection criteria, was developed. The relationship between the previous tolerance with the achievable relative RMSE and the number of strain segments was obtained (Fig. 5). These set of charts have the potential to be used as design guidelines when using the reported soft materials in robotic applications, since they highlight the trade-off between achievable accuracy and computational cost. The PL method was implemented into the linear viscoelastic models, obtaining the PL-Wiechert and PL-SLS models. These models were transformed into finite differences equations to evaluate their stress response. The obtained results demonstrated the great accuracy of the PL-SLS in describing the stress-strain curve of six soft materials. In contrast, implementing the PL method into a more complex viscoelastic model such as the Wiechert model, did not meet the expectations. The latter is due to an essential difference between Eq. 5 and Eq. 6. In the latter equation, k_i^* interacts with the strain and the strain rate whereas in the former, k_i^* only interacts with the strain. This lack of interaction allows the abrupt step changes caused by the Heaviside function to disturb the PL-Wiechert model response. In future works, we plan to explore the implementation of the PL method to the differential equation of the Wiechert model. Finally, The PL method is able to approximate complex viscoelastic and non-linear behaviors in soft materials using the linear viscoelastic models.

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